THE MATHEMATICS TEACHER

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Bernouilli (Jean)

THE MATHEMATICS TEACHER

Volume XXVI Number 8

Edited by William David Reeve

What is Wrong With Teachers of Mathematics?

THE NATIONAL COUNCIL of Teachers of Mathematics needs the individual support of two groups of teachers of mathematics who are not members. The first group consists of those teachers who, according to their own testimony, have never heard of the National Council of Teachers of Mathematics or *The Mathematics Teacher*, to say nothing of the yearbooks published annually by the Council. If anyone doubts that the situation is serious, let him travel about the country and talk to groups of teachers. It will soon be obvious that some vigorous missionary work needs to be done. Surely no meeting of mathematics teachers of any importance should be held anywhere without someone's giving at least a short description of the work that the Council is trying to do and what a teacher needs to do to become a member of the organization. If reports that come to the office of *The Mathematics Teacher* are true, many meetings of mathematics teachers are held and not one word is said about the magazine or the yearbooks, and no one is urged to join the Council. The office of *The Mathematics Teacher* will gladly send subscription blanks describing the work of the Council to anyone who is interested enough in the work we are trying to do to write us about them.

In the second place, many teachers of mathematics who know all about the work of the Council, the magazine, and the yearbooks do not take enough interest in the Council's welfare to join the

organization or to keep up their membership once they have joined. It is of small help to the Council if a teacher reads someone else's magazine or if the entire mathematics department of a high school subscribes for only one copy.

There is little chance now for the Council to save much money when the membership dues are only $2.00 No other magazine of equal value and importance can be obtained for such a small outlay of money. However, if we can double our subscription list, we could have a little extra money to help finance our organization in its attempts to solve some of the pressing problems of the day.

We need more money to help finance the work of the important standing committees of the Council which may have to be disbanded unless the classroom teachers of mathematics in this country take more interest in our welfare—yes, in their own welfare, because the problems which are pressing are closely allied to the interests of all of us.

As everyone knows, mathematics, like some of the other cultural subjects, is under fire and, while there is no immediate danger that mathematics will go as Greek went, there will be much loss to many children if we do not succeed in awakening our teachers of mathematics to the need for their supporting the only nation wide organization that is trying to fight their battles. Members of the Council and friends of mathematics everywhere should do all in their power to get teachers to join the Council in order that mathematics may retain its high place in the curriculum.

It is needless to say that nothing that was said above is in any sense a criticism of those loyal members of the Council who have done their best to advance the interests of mathematics. Their work is appreciated and the Council could not continue to function without it.

W. D. REEVE

Important Notice

The directors of the National Council of Teachers of Mathematics and a number of trustee and committee members of the Mathematical Association of America will hold a luncheon conference Saturday noon and afternoon December 30th at Cambridge, Mass. (place to be announced), to discuss various topics of mutual interest, in particular, the status of mathematics in secondary schools.

The Plane Geometry Scholarship Tests in Ohio*

By H. C. CHRISTOFFERSON
Miami University, Oxford, Ohio

There are at least three parts to the Scholarship program carried on by the State Department of Education in Ohio under the leadership of Ray G. Wood: the Every Pupil Test program in December and April of each year, the State Scholarship Contest in May, and the General Scholarship Test for High School Seniors usually given in March each year. This report is concerned with the Every Pupil Test program only and largely with the geometry test results for December 1930 and 1931. These results have been carefully diagnosed and presented in two bulletins entitled, "The Learning and Teaching of Geometry"[1] and "The Ohio Survey of Elementary Mathematics."[2] It shall be the purpose of this paper to present some of the most striking results given in those bulletins.

Both tests were planned to cover four types of activities, and to include nothing beyond the theorem concerning the sum of the angles of a triangle. The four parts of the 1930 test were, the making of simple constructions; the giving of reasons for statements in a proof; the solving of simple numerical problems which involve the concepts and facts of geometry; the proving of an "original." The 1931 test included parts on vocabulary and on equations. (See Table II) Table I gives the general summary of scores taken from a random sampling of 100 high, 100 median, and 100 low papers. The median of the 100 median papers, 1930, was 30.3 while the actual median for the entire group of 8022 pupils was 29.7, showing that the sampling was fairly representative.

An embarrassingly striking result is shown in Table I. The threefold distribution shows a large overlapping of the three groups. In 1930, twenty-nine "high" papers were nearly perfect, three were only "28–31." The "low" column for 1930, shows that in one school

* A paper presented at the Ohio State Educational Conference, Columbus, Ohio. April 17, 1933.

[1] Christofferson, H. C., Clifton, J. L. and Wood, E. R. "The Learning and Teaching of Geometry." State Department of Education, Columbus, Ohio, 1931. 48 pp.

[2] Wood, E. R. and Skinner, B. O. "The Ohio Survey of Elementary Mathematics." State Department of Education, Columbus, Ohio. Bulletin No. 24. 82 pp.

the very lowest score in the class was 32–35, which is as high or higher than the very highest score in ten other schools. These columns for high and low scores show 6 low scores overlapping 10 high scores. Generalizing from this fact, on the assumption that the sampling is representative, we can say that in six per cent of the schools of Ohio the very poorest student does work on a par with that of the very best student in ten per cent of Ohio's schools. The 1931 test shows an even greater overlapping.

TABLE I

Total Scores on the Every Pupil Test in Plane Geometry December 1930 and 1931 by High, Median and Low Pupils from 100 Schools Selected at Random

Score	Frequency in Each Group							
	1930				1931*			
	High	Median	Low	Total	High	Median	Low	Total
48–50	29			29	1			1
44–47	26			26	30			30
40–43	22	6		28	34	2		36
36–39	13	13		26	19	10		29
32–35	7	19	1	27	8	35	1	44
28–31	3	24	5	32	6	23	5	34
24–27		19	5	23	1	16	5	22
20–23		12	3	15	1	11	3	15
16–19		5	11	16		1	11	12
12–15		2	25	27		2	25	27
8–11		1	22	23			22	22
4–7			24	24			24	24
0–3			4	4			4	4
Total	100	100	100	300	100	100	100	300
Median**	45.1	30.3	12.0	30.3	67.2	54.3	36.3	53.5

* Scores scaled down from maximum of 80 to fit the same distribution as 1930 test and yet show the same overlapping as in original tables.

** Computed from a more detailed distribution.

It is a mistake to place too much stress, however, on this comparison. The purpose of the tests was not to make comparisons between schools. If comparisons are to be made seriously the tests should not only be given under exactly the same conditions, but also scored by the same scorers. Some teachers follow the key absolutely, even to the extent of marking right answers wrong, others interpret the key more generously considering that the author may not be infallible. This statement does not necessarily imply

that some teachers are dishonest; it does call attention to the two variables, the administering and the scoring of the tests, which tend to invalidate comparison of results between schools.

Table II is based on all pupils taking the first test. It shows that only 36.3% of the pupils, 1930, could succeed in solving the "originals," with the mean percent of accuracy between 60 and 70 percent for the other parts of the test. The errors in problem solving were largely computational. They show carelessness and lack of common sense judgment by such answers as 130° for an acute angle. This paper will omit detailed discussion of these errors because of lack of time, and concentrate on the errors in the other parts of the 1930 test.

A second part of the test was designed to test a pupil's ability to give reasons for statements in a proof. One proof had seven steps and the other thirteen. Each proof was given in full except that a blank was left for the reason given for each statement. From a lettered list of reasons the pupil was to select the correct reason for each statement in the proof and place its letter in the space provided. A few of the most striking errors are suggestive. In bisecting an angle ABC, after cutting the sides of the angle at A and C, arcs with A and C as centers and the same radius are described intersecting within the angle at D. Then lines AD and CD were drawn. Then $AD=CD$ because they were constructed equal. Thirty per cent of the median group gave wrong answers or omitted the reason for this statement. Eight per cent of the median papers gave as the reason, "Corresponding parts of congruent triangles are equal"; five per cent gave, "Definition: To bisect means to cut

TABLE II

Average Per cent of Correct Responses for Each Part of the Every Pupil Test in Plane Geometry December 1930 and December 1931

Parts	Average Per cent Correct	
	1930	1931
Constructions	69.5	69.3
Reasons	64.0	—
Problems	64.2	46.2
Originals	36.3	—
Symbols and Terms	—	60.9
Equations	—	44.5
Whole Test	59.4	53.7
Number pupils	6853	4897

into two equal parts" and two per cent of the median papers gave, "They are identical"; four per cent gave other absurd responses; and eleven per cent omitted the answer.

After locating point D, line BD is drawn. The statement $BD = BD$ occurs in the proof, and, following that, the statement that triangle ABD is congruent to triangle CBD, the reason being, "Two triangles are congruent if three sides of one equal respectively three sides of the other." The three pairs of equal sides had just been listed, yet thirty-four per cent of the median group failed to give the right reason for the congruence of the triangles. Twenty-one per cent of the median group gave as the reason, "side-angle-side"; five per cent gave "angle-side-angle"; and five per cent omitted the answer.

These errors seem to indicate a blind groping around for a response often used and accepted, rather than one which had a sensible application to the problem at hand. There seems to be lacking in many cases a sense of the sequence of steps. Triangles are assumed to be congruent before they are proved congruent. Angles to be proved equal are accepted as equal in the proof.

In the second proof fifteen per cent of the median group failed to recognize why $GM = GM$ and twenty-three per cent omitted it. After listing the two sides and included angle of each triangle and stating the equalities, fifty-four per cent of the median group failed to see that the triangles were congruent by side-angle-side, fifteen per cent giving as the reason the "three-sides" theorem; five per cent, the "angle-side-angle" theorem; two per cent, the utterly meaningless reason that corresponding parts of congruent triangles are equal; one per cent, that equals subtracted from equals leave equals; one per cent, that vertical angles are equal, and thirty-one per cent omitted it.

When one considers that these results are from median papers and that after three months of geometry so many pupils fail to associate the meaning of the statements with the situation to the extent indicated above, there seems to be left only one of two conclusions: either the test did not measure the pupils' real ability, or, our teaching is often such a mechanical repetition of the reasoning of others that many pupils are merely repeating words which to them are meaningless. Reasons seem to be selected on the basis of the frequency with which they have given satisfaction in the past rather than on an understanding of their meaning.

TABLE III

Per cent of Pupils Having One or More Steps Right in the Construction Problems Part III of the Every Pupil Test in Plane Geometry, December 1930. High, Median, and Low Pupils from 100 Schools Selected at Random

Problem	Number of Steps Right	Per cent in Each Group		
		High	Median	Low
—A—	First Only	97	88	35
	First Two	97	85	29
	First Three	96	82	20
	First Four	93	72	14
	First Five	88	57	8
	All Six	88	56	7
—B—	First Only	99	90	48
	First Two	91	62	18
	First Three	90	57	12
	First Four	79	38	5
	All Five	77	35	5

A third part of the test involved two constructions: the bisector of an angle *B* and the perpendicular bisector of a line segment *EF*. The completed figure was given with all construction lines on it, and below a lettered list of statements giving the various steps in the process but not in correct order. The student was to select the statements in the correct order and place their letters in a table. Table III indicates that in the median group, for problem A, eighty-eight per cent started with the correct first step; eighty-five per cent got two steps right; eighty-two per cent, three steps; seventy-two per cent, four steps; fifty-seven per cent, five steps; and fifty-six per cent, all six steps right. When one considers that in a test of this kind that there will be language difficulties and that some will not quite understand what they are to do, this result is very creditable. Yet even this simple task is very difficult for the slower pupils if we can accept these results as a criterion. Only thirty-five per cent of the "low" pupils got the first step while only seven per cent of them finished.

Problem B seemed a little more difficult. It happens, however, that 21 errors at step two consisted in omitting one rather unimportant step, namely, the naming of the points of intersection of the arcs, *G* and *H;* otherwise these 21 median papers were right. Adding 21 to the 62 who got two steps right in problem B, and almost as many to each of the other numbers would make the results for the two problems consistently high. Clearly, we seem to be

doing a fairly good job of teaching children how to make constructions. They do seem to know in large measure what to do first, second, third and fourth.

TABLE IV

Per cent of Pupils Having One or More Steps Right in the Proof of the Original Exercise in Part II of the Every Pupil Test in Plane Geometry, December 1930. High, Median, and Low Pupils from 100 Schools Selected at Random

Steps Correct	Per cent in Each Group			
	High	Median	Low	Total
1	87	50	17	51.3
1 and 2	79	25	4	36.0
1, 2, and 3	73	18	3	31.3
1, 2, 3, and 4	70	18	2	29.3

Table IV shows the success with the "original" part of the test. Only fifty per cent of the median group could get the right start, half of these lost out at the second step, and only eighteen per cent of the median pupils gave correctly the four steps which were necessary to prove the exercise. Seventy per cent of the high pupils finished correctly, but for the low pupils while seventeen per cent got one step right only four per cent got two right and only two per cent finished all four steps in the proof of the original.

In general the outstanding conclusions of this study seem to be as follows. In applying geometry principles to numerical problems our pupils do fairly well, the errors seem to be largely due to careless computation and lack of common sense checking; in making constructions they do reasonably well; but in being able to give acceptable reasons for statements they seem to do unbelieveably poor work, and when it comes to organizing their thinking so as to build up a proof for an original exercise they fail miserably. They seem to lack the ability to organize and assemble facts and ideas even more than their three months training would warrant.

What suggestions come from these conclusions for the improvement of our teaching of plane geometry in Ohio? (1) In problem solving a much greater emphasis must be placed on accuracy of computation and checking of answers. (2) Maintain and improve our fairly good work in constructions. (3) Teach geometry not only as problem solving and making constructions, but also as a science of reasoning in which meaningful reasons for statements must be given and in which sequence is of great importance.

To what shall we ascribe the two major weaknesses described above? There are several rationalizations. Some pupils have intuitive geometry in the 7th and 8th grade and they learn to make constructions and to solve many problems. In the senior high school they can then begin at once to prove, to demonstrate conclusions in geometry. Consequently these pupils have more experience in these two types of activities. Then too, some geometry teachers feel that the important part of geometry is for pupils to be able to repeat like so many parrots the reasoning of others rather than to do some reasoning for themselves and to understand fully what it is all about. Some of us train children to apply theorems to numerical problems and teach them the technique of making constructions, but place little emphasis upon the nature of rigorous proof. Furthermore, the philosophy governing our teaching in some cases makes geometry largely a glorified course in mechanical drawing, with a few problems.

The question then upon which geometry teachers need to agree is the fundamental one concerned with the possible function of geometry in our educational system. If the fundamental purpose and nature of geometry is fully understood our teaching will naturally be modified in harmony with this objective. If some teachers are teaching geometry as mechanical drawing; others as a memory course on the reasoning of mathematical geniuses; still others as a course in problem solving with no regard for logical rigor, logical sequence, or the nature of reasoning; and then still others teach it as a science of reasoning with drawing, problem solving, and a knowledge of the traditional proofs of traditional theorems as by-products, then it should be evident that the results of such varying types of teaching will vary.

In conclusion, it may be well to summarize the opinions of some of our best authorities on the function of geometry in order to concentrate our attention and interest upon the achieving of this high purpose.

The National Committee on the Reorganization of Mathematics states that the principal purposes of instruction in plane geometry are: "To exercise further the spatial imagination of the student, to make him familiar with the great basal propositions and their applications, to develop an understanding and appreciation of a deductive proof and the ability to use this method of reasoning

where it is applicable, and to form habits of precise and succinct statement, of logical organization of ideas, and of logical memory."[3]

Professor Reeve of Teachers College, Columbia University in the Fifth Yearbook of the National Council states, "The purpose of geometry is to make clear to the pupil the *meaning of demonstration, the meaning of mathematical precision, and the pleasure of discovering absolute truth.* If demonstrative geometry is not taught in order to enable the pupil to have the satisfaction of proving something, to train him in deductive thinking, to give him the power to prove his own statements, then it is not worth teaching at all."[4]

Professor C. B. Upton in the Fifth Yearbook claims that, "some teachers may at first think that our purpose in teaching geometry is to acquaint pupils with a certain body of geometric facts or theorems or with the applications of these theorems in everyday life, but on second reflection they will probably agree that our great purpose in teaching geometry is to show pupils how facts are proved. Our great aim in the tenth year is to teach the nature of deductive proof and to furnish pupils with a model for all their life thinking."[5]

Mr. W. S. Schlauch in his chapter on the analytic method in the Fifth Yearbook, asserts: "Geometry seems, of all school subjects the best adapted to initiate a student into the meaning of mathematics as a science of necessary conclusions."[6]

The statement made by Professors Birkhoff and Beatley of Harvard is interesting. "In demonstrative geometry the emphasis is on reasoning. To the extent that the subject fails to develop the power to reason and to yield an appreciation of scientific method in reasoning, its fundamental values or purposes of instruction is lessened. The bulk of the geometric facts of importance belong properly in the intuitive geometry of grades VII and VIII, and are not the chief end of our instruction in demonstrative geometry in the senior high school."[7]

[3] National Committee on Mathematical Requirements. *The Reorganization of Mathematics in Secondary Education.* Houghton Mifflin Co., 1927. p. 48.

[4] Reeve, W. D. "The Teaching of Geometry," *Fifth Yearbook of the National Council of Teachers of Mathematics.* Bureau of Publications, Teachers College, Columbia University, New York City. 1930. p. 154.

[5] Upton, C. B. "The Use of Indirect Proof in Geometry and in Life," *Ibid.* p. 131–2.

[6] Schlauch, W. S. "The Analytic Method in the Teaching of Geometry." *Ibid.* p. 134.

Professors C. H. Judd and E. R. Breslich of the University of Chicago agree that *there is no justification in algebra and geometry for mere mastery of formulas and repetition of textbook demonstrations*. . . . The duty of higher education is to direct the pupil's attention to higher forms of generalized or scientific thinking."[8]

Professor W. W. Hart from the University of Wisconsin claims that "Demonstrative Geometry uniquely develops the habit of deductive thinking. . . . The training in demonstration should come from the solution of originals, and this must be made the chief aim of the course."[9]

From England the pen of John Perry informs us that "we pay teachers to give us something that will be useful in our education and useful to us in life. . . . One use of mathematics is giving men mental tools as easy to use as their arms or legs; enabling them to go on with their education." Further, he contends that mathematics should teach "a man to think things out for himself and so deliver him from the present dreadful yoke of authority."[10]

These statements of the functions of demonstrative geometry in the tenth grade, may be summarized under two headings.

(1) *Practical, Immediate, or Direct Value.* There is no question concerning the practical value of knowing that the sides of similar triangles are proportional, or that an angle may be constructed equal to another by a definite process. Furthermore, there is also practical value in learning to make precise statements, to appreciate the need and value of definitions and postulates, to know the power and the technique of rigorous deductive proof.

(2) *Indirect, Transcendent, or Concomitant Values.* The simple concepts with which geometry deals give it a higher function. Geometry achieves its highest possibilities if, in addition to its direct and practical usefulness, it can establish a pattern of reasoning; if it can develop the ability to think clearly in geometric situations and to use the same discrimination in non-geometric situations; if

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[7] Birkhoff, G. D. and Beatley, Ralph. "*A New Approach to Elementary Geometry.*" *Ibid.* p. 86.

[8] Judd, C. H. *The Psychology of Secondary Education.* Ginn and Company. 1927. p. 111.

[9] Hart, W. W. "*Purpose, Method and Mode in Demonstrative Geometry,*" Mathematics Teacher XVII, 1924, p. 172.

[10] Perry, John *British Association Report:* "Teaching of Mathematics." Macmillan and Company. 1901. p. 45.

it can develop the ability to generalize with caution from specific cases, and to realize the force and all inclusiveness of deductive statements; if it can develop an appreciation of the place and function of definitions and postulates in the proof of any conclusion, geometric or non-geometric; if it can develop an attitude of mind which tends always to analyze situations, to understand their interrelationships, to question hasty conclusions, to express clearly, precisely, and accurately non-geometric as well as geometric ideas.

The two studies with which this paper deals tend to show a lack of emphasis upon this second or higher function of geometry as a science of reasoning. We, as geometry teachers in Ohio, seem to neglect the reasoning side of geometry in comparison with the relative excellence of our teaching in the mechanical phases of the subject.

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Supplementary Projects: Procedure

By J. S. GEORGES
Crane Jr. College, Chicago, Illinois

IN A PREVIOUS PAPER[1] the purpose of supplementary projects in mathematics was discussed. The present paper outlines a suggested procedure which teachers of mathematics may find useful in directing supplementary project work in their classes. The procedure was used successfully at the University High School, the University of Chicago, where for a number of years the author was in charge of supplementary project work in the department of mathematics.

If the individual pupil is to derive benefits from supplementary work he must be guided and supervised in the selection of the project, the reference work, the solution, and the presentation of the findings. The present outline is an attempt towards such supervision.

It is recommended that the supplementary projects which are exceptional and of general interest, be bound, cataloged, and placed in the library of the school where they may be used as reference in similar line of work. This serves a double purpose, first of encouraging the pupil engaged in supplementary project work to do superior type of work and second, to inspire others to undertake projects and to carry them through to a successful conclusion.

SUPPLEMENTARY PROJECTS IN MATHEMATICS

I. *Form of the write up of the project.*

A. *Title page.*

1. Title. Give a precise title of the project.
2. Author. The author's name should be given below the title.
3. Class and section. Give your class and section.
4. School.
5. Date.

[1] THE MATHEMATICS TEACHER, Vol. XXVI, No. 5, p. 283.

Illustration

AXIAL SYMMETRY

By

John Orr Raymond

Mathematics II, Mr. Georges, Instructor
The University of Chicago High School
November, 1930

B. *Table of contents.*

1. Main sections of the project.
2. Refer to pages in the project.

Illustration

AXIAL SYMMETRY

Table of Contents

1. Axial symmetry defined 1
2. Symmetry of regular polygons 1–10
3. Symmetry of curves 11–15
4. Symmetrically situated figures 16–20
5. Symmetrical figures in art 21–24

C. *Preface.* Give a brief statement of the project, its meaning and importance in mathematics and elsewhere. If the project is a problem for solution or construction, give an exact statement of the problem and its relation to related subjects. If it is a biography of a great mathematician, state briefly his contributions to the subject.

D. *The body of the project.*

1. The exposition should be typed, whenever possible.
2. The drawings and constructions should be on separate pages.
3. The construction page should be bordered.
4. Drawings and constructions should be made on regular drawing paper.
5. Use India ink in drawings and constructions.
6. Tables should be neatly drawn, not on separate pages, unless long, and should be bordered.

Illustration

Table I

Number of Axes of Symmetry

Polygon	Number of Sides	Number of Axes	Formula
Triangle	3	3	$N=n$
Square	4	4	$N=n$
.....	.....		

7. Formulas and equations should be printed.
8. Graphs should be on separate graph paper.
9. Large sheets should be neatly folded and attached properly.

E. *Citations.* Use footnotes. Use standard form.

Illustration

"Regular polygons of an odd number of sides have more than one axis of symmetry, but no two of them will be at right angles to each other."[2]

F. *Bibliography.* List alphabetically all the books, periodicals, and projects consulted, which have contributed to the work of the project.

II. *Procedure.*

A. *Selecting a project.* Unless you have selected a project yourself ask the instructor in charge of supplementary projects to assign you one which is the type of work in which you are interested, and are capable to carry to a satisfactory conclusion.

B. *Library work.*

1. Consult all books that have any reference to your project. Use the index of the book for direct information.
2. Ask the director of supplementary projects for exact references.
3. Ask your instructor for helpful information.
4. Ask the librarian to help you in locating the reference books, periodicals, and projects.
5. Be prepared to spend some time in departmental libraries, or the general library, if so advised.

[2] Boon, F. C. *A Companion to Elementary Mathematics*, p. 153.

C. *Preparatory work.*

1. Take notes on what you read.
2. Write down your own thoughts and reactions as you read.
3. Think about your project. Turn it over in your mind.
4. Draw your constructions and diagrams in pencil on ordinary paper first. See that they are correct before copying them in ink.
5. Arrange your construction work in definite steps. Number the steps.
6. Give reasons for every step in the proof of a theorem.
7. Prove your construction.
8. Arrange your algebraic work in steps. Number the steps.
9. Each line should contain only one step.
10. Consult the director of supplementary projects, before copying your project in final form.

D. *Presentation of the project.*

1. Go over your copy to detect mistakes in computation.
2. Go over your copy to detect misspelled words.
3. Go over your project again to make necessary revisions.
4. Present it in the form suggested.
5. When your project is completed hand it in at once to the director of supplementary projects.
6. Your copy will not be returned to you. You should have a carbon copy for yourself.
7. Your project will be used for reference either in the department, or placed in the library of the school. Take pride in it.
8. Your instructor will properly acknowledge your work by notifying the Principal's office, and through it your parents.
9. Supplementary project work is a privilege. Your project should be a testimonial of that privilege.

National Council Members!

Plan to join the directors and other members of The National Council of Teachers of Mathematics at the annual meeting at the Hotel Cleveland in Cleveland, Ohio on February 23 and 24, 1934. The tentative program of the meeting will be found on page 494 of this issue of The Mathematics Teacher.

A Study of Davies' University Arithmetic, 1846

By PAUL V. KUNKEL

This study was undertaken to try to answer the following questions: (1) What did our ancestors consider to be the content of arithmetic? (2) Was this more, or less practical than our present content? (3) Are the problems contained in the old arithmetics practical problems, and if so, might they be used as patterns for the construction of problems which would be practical today? (4) What interesting facts and information may be gleaned from the study of this text?

The first question I have tried to answer by quoting some of the problems, exercises, rules, and postulates which are contained in the text. The second question is one into which opinion enters to a great extent, and therefore, I can but try to convince my readers of the validity of my conclusion by showing some of the things contained in the volume. The third question, being somewhat similar, will be answered with the second. It is rather disappointing that the fourth question, which I consider the most valuable, cannot be definitely and precisely answered.

TO
THE TEACHERS OF THE UNITED STATES
THIS
TREATISE ON ARITHMETIC

the last of a series of works designed to lessen the labor
and improve the systems of teaching,

is
RESPECTFULLY DEDICATED
BY
THE AUTHOR

It is offered as a token of his grateful appreciation of the indulgence with which his other works have been received, and as a testimony of his regard for those with whom he has long been a co-laborer in the great work of public instruction.

A dedication of this sort is certainly ambitious. Davies had had considerable experience in the writing of different texts which included algebra, geometry, and the calculus. This, then, should be a text carefully constructed and suitable to the needs of the pupil and teacher. Evidently the pupils who came to West Point, where the author taught, had not had proper training in arithmetic, and Davies wished to remedy the defect.

Another note in his preface which is a familiar situation is his attempt to show the pupil the value of the study of arithmetic. He says this:—"It lies at the foundation of the exact and mixed sciences, and a knowledge of it is an important element either of a liberal or a practical education. While it is a science in all that concerns the properties of numbers, it is yet an art in all that relates to their practical application. It is the first subject in a well-arranged course of instruction to which the reasoning powers of the mind are applied, and is the guide book of the mechanic and man of business. It is the first fountain at which the young votary of knowledge drinks the pure waters of intellectual truth."

The following problems are of interest because of their historical data.

1. Newton (Sir Isaac) was born at Woolsthorp, a hamlet in the parish of Colsworth, in Lincolnshire, on Sunday, the 25th December, 1642; and died at Kensington, in Middlesex, on Monday, the 20th March, 1727. Euler (Leonard) was born at Basil, in Switzerland, on Tuesday, the 15th April, 1707; and died at Petersburg, in Russia, on Sunday, the 7th September, 1783. Lagrange (Joseph Louis) was born at Turin, in Italy, on Friday, the 30th January, 1736; and died at Paris, on Saturday, the 10th April, 1813. Laplace (Pierre Simon, marquis of) was born at Beaumont-en-Auge, in France, on Thursday, the 23rd March, 1749; and died at Paris, on Tuesday, the 27th March, 1827. How old was each of these eminent philosophers and mathematicians at the time of his decease? and how many years was it from the time each died to January 1st, 1846?

2. The great bell at Oxford, the heaviest in England, weighs 7 T. 11 cwt. 3 qr. 4 lb.; St. Paul's bell in London weighs 5 T. 2 cwt. 1 qr. 22 lb.; and Tom of Lincoln weighs 4 T. 16 cwt. 3 qr. 18 lb.; How much are these bells, together, inferior in weight to the great bell at Moscow, the largest in the world, which weighs 198 T. 2 cwt. 1 qr.?

3. The list of vessels in the British navy, on the first of January 1846, was as follows: sailing vessels in 'commission' and in 'ordinary,' 361; sail vessels building, 42; steam frigates, 11; steam frigates building, 8; steam packets, 25; receiving and quarantine vessels, transports, &c., 134. What is the whole number of vessels, and what is the number of each kind?

4. The number of scholars attending the public schools of Maine, in 1839, was 201024, and the amount expended for the support of the schools was $258113,43.

What was the cost to the state for the tuition of each scholar? (Notice the use of the comma as a decimal point. One of the few instances in the book.)

5. There was inspected in Onondaga county N. Y., in 1844, 4003554 bushels of salt. The duties collected on these amounted to $240305. What was the duty on each bushel?

6. A man having a horse for sale, offered it for $225 cash in hand, or $230 at 9 months, without grace; the buyer chose the latter; did the seller lose or make by this offer, supposing money to be worth 7 percent?

7. What was the population of the British provinces in North America in 1834, the population of Lower Canada being stated at 549005, of Upper Canada 336461, of New Brunswick 152156, of Nova Scotia and Cape Breton 142548, of Prince Edward's Island 32292, of Newfoundland 75000?

8. The number of troops furnished by each of the states in the revolution, was as follows: New Hampshire, 12497; Massachusetts, 67907; Rhode Island, 5908; Connecticut, 31939; New York, 17781; New Jersey, 10726; Pennsylvania, 25678; Delaware, 2386; Maryland, 13912; Virginia, 26678; North Carolina, 7263; South Carolina, 6417; Georgia, 2679. What was the number of regular troops engaged during the war?

9. St. Augustine was founded Sept. 8th, 1565. Jamestown was founded May 13th, 1607. The Battle of Princeton was fought Jan. 3rd. 1777. Cornwallis surrendered, Oct. 19th, 1781. Washington was first inaugurated April 30th, 1789: he died Dec. 14th, 1799. The French Berlin decree was issued Nov. 21st, 1806, and the British orders in Council, Nov. 11th, 1807. The United States declared war against Great Britain June 18th, 1812. The Guerriere was captured by the Constitution Aug. 19th, 1812. The frigate United States captured the Macedonian, Oct. 25th, 1812. York in Upper Canada was captured by the Americans, and General Pike killed, April 27, 1813. The British were repulsed from Sackett's Harbor by the Americans commanded by General Brown May 28th, 1813. Fort George was captured May 27th, 1813. The Battle of Lake Erie was fought Sept. 10th, 1813. The Battle of Chippewa was gained by a detachment of the American army under General Scott, July 5th, 1814. The Battle of Niagara, or Lundy's Lane, was fought July 25th, 1814. General Brown conducted the sortie from Fort Erie, Sept. 17th, 1814. The Battle of New Orleans was fought Jan. 8th, 1815. Adams and Jefferson died July 4th, 1826. The compromise bill was introduced into the senate Feb. 12, 1833. General Lafayette died May 20th, 1833. The Cherokees began to remove May 26th, 1838. What time has elapsed from the date of each of these events to March 17th, 1846.

A different type of problem, showing some of the old types of measurements will next be considered.

1. A man with his family, consisting of 5 persons, usually drink 7.8 gallons of beer in a week: how much would they drink in 23.5 weeks, if the family was to be increased by 3 persons?

2. If 12 pieces of cannon, eighteen-pounders, can batter down a castle in one hour, in what time would nine twenty-pounders batter down the same castle, both pieces of cannon being fired the same number of times, and their balls flying with the same degree of velocity?

3. If the penny loaf weighs 8 ounces when the bushel of wheat costs 7 s. 3 d., what ought it to weigh when the wheat is 8 s. 4 d. per bushel?

4. The quick step in marching is 2 paces per second, at 28 inches each: at what rate is that per hour, and how long will a troop be in reaching a place 60 miles distant, allowing a halt of an hour and a half for refreshment?

5. If I lend my friend £200 for 12 months on condition of his returning the favor, how long ought he to lend me £150 to requite my kindness?

6. A horse in his furniture is worth £52 10 s.; out of it £24 10 s. 6 d. How much does the price of the furniture exceed that of the horse?

7. From 149348761340526465 take 48973024012394

8. In 57 pieces of cloth, each 35 ells Flemish, how many ells English and nails?

9. How many barleycorns will reach round the earth, each degree being 69½ miles? and how many quarters of barley are contained in such a number of barleycorns, admitting 79922 barleycorns to fill a pint?

10. In 28 tons of round timber, how many solid inches?

11. In 27 cords of wood, how many solid inches?

12. In 30720 quarts, how many lasts?

13. How many sacks in 103 London chaldrons and 12 bushels of coal?

14. How many seconds in 6 years of 365 days 23 hr. 57 m. 39 sec. each?

15. In 77 hhds. of brandy, how many half-ankers?

16. In 38 butts of porter, how many pints?

17. Multiply 23869572491872 by 4007865347912

18. What is the sum of three hundred and forty units of the third order, seven thousand six hundred and fifty of the fourth, three millions of the second, and six trillions seven hundred and ninety-nine of the first?

New York, May 1, 1846

19. Mr. James Spendthrift
Bought of Benj. Saveall

18 pounds of tea at 85 cents per pound.....
35 pounds of coffee at 15½ cents per pound.....
27 yards of linen at 65 cents per yard.....

Rec'd payment

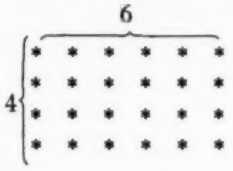
Benj. Saveall

20. Write in figures, sixty-five decillions, eight hundred quadrillions, seven hundred and fifty billions, seven hundred and fifty-one trillions, nine hundred and seventy-five thousand, three hundred and ten.

21. How large is a sheet of Demy? Of Elephant? Of Double Elephant? Of Columbia? Of Antiquarian? Of Atlas? etc. (This problem is to be solved by reference to a table of sizes of drawing paper.)

These problems show the large vocabulary necessary for our fathers to study arithmetic. They have been selected because of some peculiar fact in vocabulary, large numbers, or absurdity, or relation to problems of antiquity. Many others were met with, that I have not given.

Here is the way in which the author rationalizes multiplication:—



The author included many rules, postulates, and propositions of which the following are typical:—

Prop. 1. If the numerator of a fraction be multiplied by any number, the denominator remaining unchanged, the value of the fraction will be increased as many times as there are units in the multiplier. Hence to multiply a fraction by a whole number, we simply multiply the numerator by the number.

Prop. 2. If the numerator of a fraction be divided by any number, the denominator remaining unchanged, the value of the fraction will be diminished as many times as there are units in the divisor. Hence, a fraction may be divided by a whole number by dividing the numerator.

Prop. 3. If the denominator of any fraction be multiplied by any number, the numerator remaining unchanged, the value of the fraction will be diminished as many times as there are units in the multiplier. Hence, a fraction may be divided by any number, by multiplying the denominator by that number.

Prop. 4. If the denominator of a fraction be divided by any number, the numerator remaining unchanged, the value of the fraction will be increased as many times as there are units in the divisor. Hence, a fraction may be multiplied by a whole number, by dividing the denominator by that number.

Prop. 5. If both terms of a fraction be multiplied by the same number, the value of the fraction will remain unchanged.

Prop. 6. If both terms of a fraction be divided by the same number, the value of the fraction will remain unchanged.

"The character 0 is used to denote the absence of a thing. As, if we wish to express by figures that there are no apples in a basket, we write, the number of apples in the basket is 0. The nine other figures are called *significant figures*, or *digits*.

The student was taught to prove addition by adding downward or by means of partial sums. He memorized certain cases and thus worked problems. Before starting the real work in arithmetic, he had to learn:

Reduction of vulgar fractions—6 cases
Reduction of denominate fractions—4 cases
Addition of vulgar fractions—3 cases
Subtraction of vulgar fractions—3 cases
Multiplication of vulgar fractions—2 cases
Division of vulgar fractions—2 cases.

Then there remained 37 tables to be memorized by the pupil. After that came long complicated rules for doing square root and cube root. When this had been accomplished the real work began by considering such topics and subjects as the following: circulating decimals and various forms of repetends; rule of three; double rule of three; practice rule; tare and tret; draft; suttle; equation of payments; double fellowship; alligation medial; alligation alternate (three cases); custom house business; tonnage of vessels; gauging; life insurance; endowments; annuities; exchange; value of certain coins in U. S. money and English; simple and compound arbitration; chain rule; duodecimals; evolution by figure; promiscuous questions; and book-keeping; in addition to all that is taught in the present day arithmetic.

I wonder whether people knew arithmetic when they were finished with this book? I feel that one could really compute after going through work as that contained in this book. Did arithmetic mean anything to the student? I don't know but personally I feel that the study of the subject must have been a dreadful bore. Here was a subject to discipline the mind if there ever was one. Here was something that would give to the student the patience of Job. But, in a larger sense, the educated man of those times was supposed to be a scholar who was familiar with many fields and who was proficient in fields like the "three r's." To this end then, a thorough and rather involved course in arithmetic would seem necessary.

"WOULD YOU be content with a materialistic explanation of this church building? It is made of the chemical elements but there is more to be said than that. It is mathematical. The very essence of its structure and coherence is mathematics, and mathematics is mental. And there is beauty here and beauty is spiritual. And week after week human souls who know they cannot live by bread alone lift up here their aspirations toward that which seems to them divine. But these same factors are in the universe at large. Mathematical structure, creative beauty, aspiring spiritual life—we cannot sum all that up in a materialistic formula. Plus ultra!"—Quotation from a sermon by DR. HARRY EMERSON FOSDICK (Pastor of the Riverside Church, New York City) on November 13, 1932, "Some Cannot Find God; Others Cannot Escape Him."

More Than One Mystery*

By CELIA A. RUSSELL
Junior High School, Winchester, Mass.

CHARACTERS: Uncle David, Helen, Mary, Rebecca, Bob, Jimmy, Carl.

STAGE SETTING: One chair—Small table with box of six apples on it.

Prologue

Song and dance to the tune of "Wearing of the Green" by 5 boys and 5 girls.

Oh, students dear, and have you heard the news that's going round?
We know the tables, twos to tens, in them we're safe and sound
We know besides a little more: some rules and a formula
And likewise why in troublous times some stocks are not at par.
We like to add and multiply, to find a width or length
But best of all an unknown X, that shows our greatest strength.

Refrain:

Oh, students dear, and have you heard the news that's going round
We know the tables, twos to tens, in them we're safe and sound.

CURTAIN RISES

Uncle David is sitting in chair reading book. The crowd rushes in onto the stage and surrounds Uncle David, crying, "Oh, tell us a story, tell us a story!" Uncle David looks over his glasses and says, "I'm busy."

HELEN: Oh, but just tell us one story, please.
U.D.: But I haven't any story tonight. I'm reading a book.
HELEN: Read us a story then.
U.D.: But you wouldn't be interested. This book is a history of arithmetic.
REBECCA: No, that does sound sort of dull, but haven't you anything interesting, any stories? Please.
U.D.: No, no stories—might be a puzzle or two.

* With acknowledgments to Dr. David Eugene Smith for the inspiration necessary to write the play.

BOB: Are they very hard, wouldn't we like those?

U.D.: Well, you might, but I'm busy.

JIMMY: Oh, just one puzzle.

U.D.: All right, then. I'll make a bargain with you. I'll tell you a puzzle and if you can't solve it then you'll have to run away and leave me alone.

CROWD: Try us, try us.

U.D.: There were once 3 apples like these. Two fathers and two sons divided those apples among themselves each receiving one apple.

CARL: Why, that couldn't be, Uncle David, there were four people.

U.D.: Oh, yes it could. I'm playing fair with you. I'm giving a puzzle that can be solved.

CARL: Well, I don't see how. Do you mean they each got a whole apple?

U.D.: Yes, each got exactly one apple, and there were two fathers and two sons there.

(*pause*)

Well, I'll give you a hint. Have you ever been to a family reunion?

BOB: Oh, I see, Uncle David, I see. My grandfather, dad and I make only 3 people. Dad is grandfather's son and I am dad's son, making two sons. And dad is my father and grandfather is dad's father—making two fathers.

CARL: Is that right? I don't see how that works.

(*U. D. smiles*)

BOB: Of course it's right. Here, I'll show you. (*And Bob puts it on blackboard*)

MARY: Well, didn't we get that, Uncle David? So that means we get another one.

U.D.: Well, as long as we're on the subject of apples—here are six apples. See if you can divide these so that each of you has one, and there's one left in the box.

MARY: Oh, that's easy.

(*Uncle David presents the box of apples to Mary. Mary takes the apples and gives one apiece to each of the children and takes one herself*)

U.D.: But the box is empty.

HELEN: Oh, I see—give me the box (*Helen puts her apple in the box*) There!

(*U. D. smiles*)

MARY: Is that it?

HELEN: Yes, see, give me the apples back and I'll show you.

(*She demonstrates*)

JIMMY: Oh, that's too easy—give us a harder one.

U.D.: So you want a hard one do you? How many of you can show that 1/2 of 8 is zero?

REBECCA: We can all do that. I'll show you. (*Blackboard—Erase top of 8 and 0 is left.*)

BOB: You could erase the lower part of the 8 just as well.

U.D.: I thought you could do that, but can you show that 1/2 of 11 is 6.

HELEN: Why, it isn't, half of 11 is 5 1/2.

CARL: Oh, it is, see—(*Goes to blackboard and puts on Roman numeral XI.*) Here it is. Erase the lower half of XI and and you have V and I left. But VI is 6 and so half of eleven is six.

BOB: But I didn't know we could use Roman numerals.

JIMMY: Oh, Uncle David, I've got one I'll bet you can't do.

U.D.: What's that?

JIMMY: Can you take ten from forty and get 50?

U.D.: Is this right? (*Goes to board and solves it. XL-takes X away and has left L*)

That isn't hard enough for me.

Here's a sticker for everyone. A farmer wanted to plant 10 trees 4 in a row. How many straight rows with 4 trees in a row could he get?

BOB: Why 4 goes into 10 2 1/2 times. He could just have 2 whole rows.

CARL: Well, I can get three. Can you put them this way, Uncle David?

(*Carl at blackboard puts*

U.D.: The puzzle said straight rows—so that's all right.

REBECCA: I can get 5 rows. But I need the trees to show you. Wait a minute. (*Rebecca summons the tree dancers. They come in with triangular trees on them and do a short dance ending up as a star. Boys on outside, girls on inside. At same time, Rebecca is drawing a picture of it on blackboard.*)

U.D.: Here's something for you to do. Each of you think of a small number less than 40, but don't tell me what it is. Multiply the number by 2, add 12, add the number, divide by 3, subtract the number. I can tell you what it is. Four. Did everyone get four?

REBECCA: I didn't.

U.D.: Did you follow directions? What number did you take?

REBECCA: 24

U.D.: (*Uses blackboard to illustrate*) Let's try it again and see if you don't get four. All ready? Your number was 24 Rebecca. First thing, multiply by 2—48. Next add 12—60. Next add the number you started with—84. Then divide by 3—28. Subtract the number you started with —4.

BOB: Oh, look out there! (*Fox, duck and cabbage waddle onto stage—river marked off.*)

U.D.: Yes, didn't you know, the farmer wants to get the fox, the duck and the cabbage across the river but he can take only one at a time. He can't leave the fox and duck together because the fox would eat the duck and he can't leave the duck and the cabbage together because the duck would eat the cabbage. How is he going to get all three across.

(*Farmer shows incorrect solution by taking the fox across first—when he returns the duck is alone because the duck has eaten the cabbage. Farmer next takes cabbage, duck is eaten by fox. Then farmer takes duck across goes back for fox and brings him over. When farmer returns on his third trip with the cabbage, duck is gone, eaten by fox.*)

(CORRECT SOLUTION: *Farmer takes duck, then fox, takes duck back when he gets fox across, leaves duck on first side, takes cabbage over and leaves it with fox and last goes back for duck.*)

CURTAIN

Epilogue

SINGERS AND DANCERS OF PROLOGUE:

Oh, scholars dear, and did you hear the fun that we have made,
The puzzles that we solved for you while we were on parade;
We met with Uncle David and we took him by surprise,
But he opened up his bag of tricks and after some surmise
We showed him oddly planted trees that grew in funny rows,
And numerals with which to do whatever we propose.

Oh, scholars kind, and did you mind the fun that we have shown,
The apples that we dared not eat till we are all alone;
We showed you how three apples often do the work of four,
Though we confess we'd sooner far that you would serve us more;
And aren't you glad to see that fox and duck and cabbage too
Have landed now on t'other side and with no help from you?

BACK NUMBERS OF THE MATHEMATICS TEACHER

The following issues of the *Mathematics Teacher* are still available and may be had from the office of the *Mathematics Teacher*, 525 West 120th Street, New York.

Vol. 14 (1921) Jan., Feb., April, May.
Vol. 16 (1923) Feb., May, Dec.
Vol. 17 (1924) April, May, Dec.
Vol. 18 (1925) April, May, Nov.
Vol. 19 (1926) May.
Vol. 20 (1927) Feb., April, May, Dec.
Vol. 21 (1928) Mar., April, May, Nov., Dec.
Vol. 22 (1929) Jan., Feb., Mar., April, May, Nov., Dec.
Vol. 23 (1930) Jan., Feb., Mar., April, May, Nov., Dec.
Vol. 24 (1931) Feb., Mar., April, May, Oct., Dec.
Vol. 25 (1932) Jan., Feb., Mar., April, May, Oct., Nov., Dec.
Vol. 26 (1933) Jan., Feb., Mar., April, May, Oct., Dec.

Price: 25c each.

Mathematics Exhibit — World's Fair, Chicago

By LILLIAN MOORE
Far Rockaway High School, New York City

THE MATHEMATICS EXHIBIT at the Century of Progress Exhibition in Chicago during the summer of 1933 consisted of two sections: A) an immense octagonal prism, on the lateral faces of which were notations concerning the various subdivisions of mathematics together with slides, illustrating the development and history of algebra and geometry; B) a collection of nineteen exhibits, illustrating various interesting mathematical principles and practical applications of mathematics.

(A) In discussing the huge octagonal regular prism in the center of the rotunda of the Hall of Science, I shall endeavor to reproduce the information given on the eight lateral faces.

(1) Arithmetic and Algebra

Theory of Numbers	Illustration
Theory of Equations	Magic Square
Groups	4 9 2
Invariants	3 5 7
Universal Algebra	8 1 6

(2) Mathematics is the science which draws necessary conclusions.—BENJAMIN PIERCE, 1870.

On this face of the prism the ballopticon tossed up a series of slides, illustrating the history of arithmetic from the knowledge of the Greeks, Romans, Hindus, Arabs, Mayas and Peruvians to the printing of textbooks in the vulgar tongue by Europeans in the thirteenth century.

(3) Applied Mathematics

Celestial Mechanics—Dynamics of Rigid Bodies—Elasticity—Hydrodynamics—Electrodynamics—Conduction of Heat—Thermodynamics—Potential Theory—Relativity—Atomic Structure—Quantum Mechanics—Statistics.

(4) Axioms of Geometry are of the nature of physical laws, to be tested by experience and to be regarded as true only within the limits of error of observations.—BÔCHER, 1894.

The ballopticon slides on this face of the prism illustrated the history of algebra. These slides showed Arabic knowledge, European translations of Arabic knowledge, the algebra used in thirteenth century Europe, later European algebra, the first algebra textbook, the solution of cubic and biquadratic equations, algebraic symbolism, English and American textbooks.

(5) Analysis

Differential Calculus	Illustration
Integral Calculus	Sphere inscribed
Functions of Real Variables	in Cylinder
Functions of Complex Variables	
Differential Equations	
Integral Equations	
Calculus of Variations	
Calculus of Probability	
Aggregates in Analysis	

(6) If we have a certain class of objects and a certain class of relations, and if the only questions which we investigate are whether ordered groups of these objects do or do not satisfy the relations, the results of the investigation are called Mathematics.—SIR ALFRED KEMPE, 1890.

This face contained a slide on the anticipation of Integral Calculus by Archimedes.

(7) Geometry

Plane—Solid—N-dimensional—Non-Euclidean—Analytic—Projective—Differential—Analysis Situs—Vector Analysis. Two curves, the Witch of Agnesi and a hypocycloid, were illustrated on this face.

(8) Pure Mathematics consists exclusively of deductions "by logical principles from logical principles."—BERTRAND RUSSELL.

On this face of the prism the ballopticon slides illustrated the history of geometry.

(B) In the following discussion of the additional mathematical exhibits, I wish to acknowledge my indebtedness to Dr. D. H. Lehmer for his aid during my hurried tour of inspection.

(1) Johansson's Gauges

Gauges of various types were arranged in a glass case. A sine bar, straight edge, length gauge, and gauge blocks having an accuracy at 68° Fahrenheit of 2 or 4 millionths of an inch per inch were on display.

(2) Mathematical Recreations

An interesting exhibit was the Mystic Number Machine, which obtained an unknown number through the application of magic squares. The booklet on Fascinating Figure Puzzles compiled by the Burroughs Adding Machine Company and distributed by them in the General Exhibits Building will be of inestimable value to faculty advisers of high school mathematics clubs.

(3) Pollock's Models

Models of the following mathematical figures were constructed by stretching strings on frames:—a twisted sextic showing four nodal double points, a space quartic with an isolated double point, quartic of the second species, twisted cubic, space octic, twisted nonic, hyperboloid of one sheet, quartic curve, space quintic, conoid, right circular cone.

(4) Skew Curve Projection

This exceedingly clever illustration of curve projection was obtained by intersecting a ruled surface with a surface of light. A quartic space curve, conic sections, and cuspidal space of degree 12 were among the illustrations.

(5) Centripetal Force

This exhibit gave an illustration of pulling toward the center.

(6) Applied Centripetal Force

The separation of liquids was used in the second illustration of centripetal force.

(7) Galton Quincunx

The normal frequency curve was illustrated in the seventh exhibit.

(8) Computation of Pi

The derivation of pi as a problem in the theory of probability followed.

(9) Ellipsograph

The ninth model showed by means of confocal conics variations in the curve of the equation: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

(10) Mathematical Machines

The machine which attracted the most attention was the Harmonic Motion Apparatus, whereby two harmonic motions could be graphically compounded. Other machines exhibited were Michelson's Harmonic Analyzer and Synthesizer in connection with Fourier Analysis, Leonardo de Vinci's astrolabe, Gonella's planimeter, Lehmer's machine illustrating the factorization of a number.

(11) Gyro-Compass Repeater

No explanation of this exhibit is needed.

(12) Trigonometry as used to fix a ship's position at sea.

The twelfth model illustrated dead reckoning and current reckoning in navigation.

(13) Rotational Momentum

This model allowed personal experimentation in obtaining rotational momentum.

(14) Gyroscopic Action of Molecules

The Barnett Gyromagnetic Effect showing magnetization by rotation was illustrated.

(15) Time and Clocks

This exhibit contained numerous clocks of various periods and nautical instruments of time. As a Philadelphian I deplored the absence of a Rittenhouse clock.

(16) Radio Exhibit

The United States Navy prepared a comprehensive radio exhibit. The Italian Government sent an exhibit of instruments used by Marconi in his wireless experiments.

(17) Gunnery

The science of gunnery as another practical application of mathematics was shown in the seventeenth exhibit. Visitors did not hesitate to use the two telescope range finders set up on the balcony.

(18) Aerodynamics

Models in a wind tunnel illustrated the application of Bernoulli's theorem to aerodynamics.

(19) Pantographer

The last mathematical exhibit was an excellent example of a large pantographer, by means of which maps were drawn to scale.

The teacher who visited this Mathematics Exhibit will have a wealth of material to use in his mathematics classes. The class in Intermediate Algebra will be intensely interested in the ellipsograph after considering the unit on graphs of quadratic equations. The derivation of pi as a problem in the theory of probability will interest pupils in plane geometry, who have just obtained pi in a different manner. A description of the twelfth model whereby the latitude and longitude of a ship at sea were determined will interest the trigonometry class. Since astronomy is as closely related to mathematics as physics or navigation, a description of the Adler Planetarium and the "Drama of the Heavens" will interest every mathematics class.

Jean Bernoulli

Born at Basel, August 7, 1667
Died in Basel, January 1, 1748

THE BERNOULLI FAMILY presents an interesting study in the inheritance of ability in mathematics. At least nine members of the group were conspicuous for this quality. In the first generation of these prominent scholars were Jacques Bernoulli (1654–1705) professor of mathematics at Basel, and Jean (1667–1748) professor at Gröningen (1695–1705) and succeeding his brother Jacques at Basel (1705–1748). Nicholas Bernoulli (1687–1759) a nephew of these brothers, taught mathematics at Padua in the position once held by Galileo, and later returned to Basel. Of the children of Jean Bernoulli, three were noteworthy: Nicholas II,[1] Daniel I and Jean II. Nicholas (1695–1726) was professor of law at Berne from

[1] In referring to the members of this family, those having the same name are distinguished by numbers. Thus the first Jacques is Jacques I, etc.

1723 to 1725 when he and his brother Daniel (1700–1782) became professors of mathematics in the Academy at St. Petersburg. Nicholas was drowned in the first year of this joint appointment, and Daniel moved to Basel in 1733. According to the family custom, he became professor in the University of Basel. A third brother, Jean II (1710–1790), studied law and later became professor of mathematics at Basel. Jean II had three sons whose careers should be noted: Daniel II (1751–1834) Jean III (1744–1807) astronomer royal at Berlin at the age of nineteen, and Jacques II (1759–1789) interested in experimental physics at Basel and later professor of mathematics at St. Petersburg. In the fourth generation, but of less importance than the preceeding members of the family was Christoph (1782–1863) son of Daniel II. Of minor importance was Jean Gustav (1811–1863), son of Christoph.

The most famous of the group were Jacques I of whom a note appeared in the *Mathematics Teacher* for October, Jean I the subject of this sketch, and Daniel I.

Nicholas Bernoulli, the father of the two famous brothers, had intended that the elder become a theologian and the younger a merchant. In spite of this, Jean Bernoulli does not seem to have considered his father's plans and he hesitated between medicine and literature, but finally turned to mathematics. He spent some time in France where he met the astronomer royal Cassini and where he tutored l'Hospital in the calculus. In the introduction to the *Analyse des infiniment petits* (1696), l'Hospital refers to him affectionately as the "young professor at Gröningen," Jean Bernoulli's appointment there having been made within the past year.

Cajori says of Jean Bernoulli[2] that

> He was one of the most enthusiastic teachers and most successful original investigators of his time. He was a member of almost every learned society in Europe. His controversies were almost as numerous as his discoveries. He was ardent in his friendships, but unfair, mean, and violent toward all who incurred his dislike—even his own brother and son.

Jean Bernoulli treated trigonometry as a branch of analysis and was perhaps the originator of the idea of trigonometric functions. He was in fact, one of the first to define a function, saying that a function of a variable quantity is a quantity composed in any manner whatsoever of this variable quantity and of constants.[3]

[2] *History of Mathematics*, p. 222.

[3] On appelle ici fonction d'une grandeur variable, une quantité composée de quelque manière que ce soit de cette grandeur variable et de constantes." *Ibid.* p. 211.

He discovered a method of finding the limiting value of a fraction whose terms tend toward zero, this being published by l'Hospital.

In June 1696, Jean Bernoulli published a challenge in the *Acta Eruditorum* which was edited by Leibniz. As this challenge illustrates a device popular in the early days of the calculus, and indeed before, it is interesting to repeat parts of the challenge here.

NEW PROBLEM

Which Mathematicians Are Invited to Solve

If two points A and B are given in a vertical plane, to assign to a mobile particle M the path AMB along which, descending under its own weight, it passes from the point A to the point B in the briefest time.

To arouse in lovers of such things the desire to undertake the solution of this problem, it may be pointed out that the question proposed does not, as might appear, consist of mere speculation having therefore no use. On the contrary, as one would readily believe, it has great usefulness in other branches of science such as mechanics. Meanwhile (to forestall hasty judgment) [it may be remarked that] although the straight line AB is indeed the shortest between the points A and B, it nevertheless is not the path traversed in the shortest time. However the curve AMB whose name I shall give if no one else has discovered it before the end of this year, is one well known to geometers.[4]

Six months later, another note appeared:

PROCLAMATION

Made Public at Gröningen, (Jan.) 1697

Jean Bernoulli public professor of mathematics pays his respects to *the most acute mathematicians of the entire world*.

Since it is known with certainty that there is scarcely anything which more greatly excites noble and ingenious spirits to labors which lead to the increase of knowledge than to propose difficult and at the same time useful problems through the solution of which, as by no other means, they may attain to fame and build for themselves eternal monuments among posterity; so I should expect to deserve the thanks of the mathematical world if, imitating the example of such men as Mersenne, Pascal, Fermat, . . . and others who have done the same before me, I should bring before the leading analysts of this age some problem upon which as upon a touchstone they could test their methods, exert their powers, and, in case they brought anything to light, could communicate with us in order that everyone might publicly receive his deserved praise from us.

The fact is that a half a year ago in the June number of the *Leipzig Acts*[5] I proposed such a problem whose usefulness linked with beauty will be seen by all who successfully apply themselves to it. . . . Only the celebrated Leibniz who is so justly

[4] See translation by Dr. Lincoln La Paz in the *Source Book in Mathematics*, pp. 645–655.

[5] An alternative name for the *Acta Eruditorum*.

famed in the higher geometry has written me that he has by good fortune solved this, as he himself expresses it, very beautiful and hitherto unheard of problem; and he has courteously asked me to extend the time limit to next Easter in order that in the interim the problem might be made public in France and Italy and that no one might have cause to complain of the shortness of time allotted.

In May 1697, the solution appeared, prefaced by the following remarks:

Up to this time so many methods which deal with *maxima* and *minima* have appeared that there seems to remain nothing so subtle in connection with this subject that it cannot be penetrated by their discernment—so they think who pride themselves either as the originators of these methods or as their followers. Now the students may swear by the word of their master as much as they please, and still, if they will only make the effort, they will see that our problem cannot in any way be forced into the narrow confines imposed by their methods, which extend only so far as to determine a *maximum* or *minimum* among given quantities finite or infinite in number. Truly where the very quantities which are involved in our problem, from among which the maximum or minimum is to be found are no more determinate than the very thing one is seeking—this is a task, this is difficult labor.[6] Even those distinguished men, Descartes, Fermat, and the others, who once contended as vigorously for the superiority of their methods as if they fought for God and country or in their place now their disciples must frankly confess that the methods handed down from these same authorities are here entirely inadequate.

With justice we admire Huygens because he first discovered that a heavy particle falls down along a *common cycloid* in the same time, no matter from what point on the *cycloid* it begins its motion. But you will be petrified with astonishment when I say that precisely this *cycloid*, the *tautochrone of Huygens* is our required *brachistochrone*.

Bernoulli's solution follows in the *Source Book*. Newton solved the problem soon after receiving it and published his work anonymously, but Jean Bernoulli claimed to recognize "ex ungue leonem."

One further point is of interest to high school mathematics. In 1712–13, Jean Bernoulli and Leibniz were in correspondence regarding the logarithm of negative and imaginary numbers. Leibniz maintained that these numbers had no logarithms. Bernoulli argued that since $dx:x=-dx:-x$, then by integrating $\log(x)=\log(-x)$, and accordingly the curve $y=\log x$ has two branches which are symmetric to the y-axis, and thus negative numbers *would* have logarithms.

VERA SANFORD

[6] Evidently a quotation from Virgil "Hic opus, hic labor est."

Vevia Blair

Miss Vevia Blair head of the Mathematics Department of the Horace Mann School for Girls in New York City died at the home of an uncle in Louisville, Kentucky on July 9, 1933. She was 62 years old.

Miss Blair was an outstanding teacher, but above all she was noted for her unusual interest in the welfare of the pupils whom she taught. She not only gave a great deal of her time and energy to her pupils while they were in school, but also kept in close touch with many of them after they were in college. Her loss will be greatly felt by such pupils and also by the school whom she served so long and so faithfully.

Miss Blair was a member of the National Committee on Mathematical Requirements which made the epoch making report on "The Reorganization of Mathematics in Secondary Education," and was individually responsible for Chapter 9 of that report on "The Present Status of Disciplinary Values in Education." She was also a collaborator with others in "The Day Junior Mathematics," having written the third book of the series herself.

Miss Blair was active in all efforts to improve the teaching of mathematics and at the time of her death had practically completed a report on an experiment in teaching the calculus to Horace Mann girls. She was a member of the National Council of Teachers of Mathematics and several other local organizations of mathematics teachers.

Harry English

Harry English, a former member of the board of directors of the National Council of Teachers of Mathematics and retired chief examiner of the public schools of Washington, D. C. died at his home in McLean, Va. on June 8, 1933. He was 67 years old.

Mr. English was appointed to teach mathematics in the Washington High School in 1887. In 1906 he was made head of the mathematics department at Central High School where he taught until 1924 when he was made chief examiner of the board of examiners whose function it is to rate all teachers for appointment within the system. He was retired two years ago after a series of strokes.

Mr. English was president of the High School Teachers Association, and for many years was a State director of the National Education Association and the delegate for the N.E.A. from the District of Columbia. He was very active in the work of the National Council of Teachers of Mathematics as long as he was able to attend meetings.

Mr. English is survived by his wife, Mrs. Margaret English, and a son Harry English, Jr.

Report of the Policy Committee

By J. O. Hassler
University of Oklahoma, Norman, Okla.

A brief statement of the purpose of the Policy Committee was printed in this journal in the issue of November, 1932, under the title, "A Tentative Program of the Policy Committee."

In pursuance of the objectives mentioned in the "Tentative Program" the Committee plans to prepare, as soon as it can be properly done, a volume designed to give the best possible modern outlook on the teaching of mathematics. Treatment of certain topics in the *Report of the National Committee* will be brought down to date. To some extent, therefore, our publication will be a supplement to that famous report. The present plans are for the Policy Committee to publish its material in the Yearbook for 1936. The Committee has arranged the following tentative outline of subject matter for the proposed publication.

I. The Nature of Mathematics and Its Place in Education

1. Mathematics as a Science
2. Contribution of Mathematics to Civilization
3. The Function of Mathematics in the Development of the Individual
4. Misconceptions of Mathematics
5. Transfer of Training and its Relation to Mathematics

II. Methods of Teaching Mathematics

1. General Objectives
2. Methods: (a) Changing methods to fit modern educational theory; (b) Successful classroom methods used by good teachers (Many illustrations of actual classroom procedures to be contributed by teachers in service showing how to introduce new topics, care for individual needs, diagnose class and individual learning, measure the use of illustrative materials, and *arouse and hold the interest of the class.*); (c) Teaching for transfer; (d) Emphasizing the function concept; (e) How to use historical material

3. Modern Tests (objective or otherwise, school and extramural) (Their value and their weaknesses, sapping the teachers independence, promoting mechanization, tending—the extramural kind—to perpetuate non-essential and practically obsolete material.)

4. Workbooks

5. The Recreational Side of Mathematics: (a) Mathematical clubs and programs; (b) Bibliography for pupils

6. Illustrations of Practical Values in Mathematics

III. The Growth of the Teacher

1. The Contribution of Higher Mathematics

2. The Contribution of Modern Educational Theory

3. Credo for Teachers

4. Bibliography for Teachers (An extension of Chapter XVI. *Report of the National Committee*, bringing the material down to date, with Possibly longer summaries—even abstracts—of some of the most important articles.)

5. Promoting Research

The first item in the outline, "Mathematics as a Science" is to be an article from the pen of Professor E. T. Bell, author of *The Queen of the Sciences*. The second, "Contribution of Mathematics to Civilization," will be written by Professor David Eugene Smith. Some of the other topics may be treated in signed articles, but most of them will be developed through the cooperative efforts of many teachers. To make this material most valuable we need the help of hundreds of interested teachers, especially in arranging the sections on classroom procedures and bibliographies. Volunteers are wanted. Write to the chairman, or any other member of the Committee, if you can help.

The outline as arranged may omit some important item. The Committee will welcome any suggestion for revision of the outline.

The following sub-committees, with chairmen as named, are being organized to facilitate the work: Committee A: Transfer of Training, Chairman yet to be selected; Committee B: Improved Methods, Mr. Harry C. Barber; Committee C: Tests and Workbooks, Dr. W. O. Shriner; Committee D: Recreations in Mathematics, Miss Ruth Lane; Committee E: Bibliography for Teachers, Mr. J. A. Nyberg.

The Policy Committee has been enlarged up to the present to include twenty-six members. Others may be added later.

MEMBERS OF THE POLICY COMMITTEE

Mr. Harry C. Barber, 22 Garden St., Boston
Prof. E. T. Bell, California Institute of Technology, Pasadena
Wm. Betz (ex officio) 208 N. Goodman, Rochester, N. Y.
Prof. C. C. Christofferson, Miami University, Oxford, Ohio
Professor John R. Clark, Lincoln School of Teachers College, Columbia University, New York City
Miss Elizabeth Dice, Vickery, Texas, Box 92 (North Dallas High School)
Mrs. H. L. Garrott, Box 427, University Station, Baton Rouge, La.
Miss Marie Gugle, Assistant Supt. of Schools, Board of Education, Columbus, Ohio
Mr. Maurice L. Hartung, University High School, Madison, Wis.
Professor J. O. Hassler (Chairman), University of Oklahoma, Norman, Okla.
Prof. E. R. Hedrick, 405 Hilgard Ave., Los Angeles, Calif. (Univ. of Calif. at Los Angeles)
Prof. G. H. Jamison, State Teachers College, Kirksville, Mo.
Prof. Cassius J. Keyser, Columbia University, New York City
Miss Ruth Lane, University High School, Iowa City, Iowa
Miss Selma Lindell, 809 E. Kingsley, Ann Arbor, Mich. (University High School)
Prof. C. N. Moore, 219 Woolper Ave., Cincinnati, Ohio (University of Cincinnati)
Prof. R. L. Morton, Ohio University, Athens, O.
Mr. J. A. Nyberg, 8248 Ingleside Ave., Chicago, Ill. (Hyde Park High School)
Miss Ruth Olson, Washburn High School, Minneapolis, Minn.
Prof. W. D. Reeve, 525 W. 120th St., New York City
Dr. Vera Sanford, State Normal School, Oneonta, N. Y.
Prof. W. O. Shriner, Indiana State Teachers College, Terre Haute, Ind.
Prof. D. E. Smith, 501 W. 120th St., New York City
Mr. R. R. Smith, Springfield High School and Junior College, Springfield, Mass.
Miss M. Bird Weimer, 614 N. Topeka Ave., Wichita, Kans. (Wichita High School North)
Mr. J. E. Whistler, Chaffey Union High School, Ontario, Calif.

TENTATIVE PROGRAM FOR THE FIFTEENTH ANNUAL MEETING OF THE NATIONAL COUNCIL

HOTEL CLEVELAND, CLEVELAND, OHIO, FEBRUARY 23 AND 24, 1934

General Theme: The Present Crisis in Secondary Mathematics

Friday, February 23, 1934

10:00 A.M.—Business Meeting of the Board of Directors.

12:30 P.M.—Director's Luncheon

2:00 P.M.—*First General Meeting*

Topic: The Future of Geometry in the High School

Speakers:

1. Professor Ralph Beatley, Harvard University, on behalf the geometry committees of the National Council.
2. Mr. Rolland R. Smith.
3. Discussion.

Friday Evening

6:00 P.M.—Informal Supper of the Delegates representing the Affiliated Organizations, followed by Reports of the Delegates.

8:00 P.M.—*Second General Meeting.*

Address of Welcome. (Speaker to be announced.)

A Mathematical Symposium, on the General Theme of the Meeting. Presented by members of the National Council, arranged and directed by Miss Mary A. Potter and assistants.

Saturday

9:00 A.M.—Annual Business Meeting.

10:00 A.M.—*Third General Meeting.*

Topic: The Problems of Ability Grouping and of Differentiated Curricula.

1. *Some Aspects of the Administrative Phases of this Problem.* Dr. C. N. Stokes, Temple University, Philadelphia
2. *Remedial Work in Arithmetic, A Challenge and a Warning Signal.* Miss Genevieve L. Skehan, Whitney School, No. 17, Rochester, N. Y.
3. *The Problem of Individual Differences and a Suggested Attempt for its Solutions.* Miss Clara D. Murphy, Evanston Township High School, Evanston, Ill.
4. *An Experiment in Teaching Graphs.* (An illustrated demonstration.) Mrs. Wilimina E. Pitcher, Rawlings Junior High School, Cleveland, Ohio.

2:00 P.M.—*Fourth General Meeting.*

Topic: What Can We Do to Meet the Challenge of the Present Situation in Secondary Mathematics?

Speakers:

1. Professor W. D. Reeve, Teachers College, Columbia University.
2. Discussion. A group of four-minute addresses. (Speakers to be announced.)
3. Professor J. O. Hassler, The University of Oklahoma, on behalf of the Policy Committee.

Saturday Evening

6:00 P.M.—*Banquet and Final Meeting.*

Greetings from Guests of Honor.

Address: Mathematics and Music Professor Carl A. Garabedian, St. Stephen's College, New York

NEWS NOTES

MATHEMATICS SECTION MEETINGS as a part of the programs for the Nebraska State Teachers Association were held October 26 and 27 in each of the six districts as follows.

DISTRICT NUMBER ONE AT LINCOLN

Demonstrations of Junior-Senior High School Work. (Whittier Junior High, 22nd and Vine Streets, Thursday, 1:40 P.M.): Algebra I or II—Room 303. Mrs. Gladys Van Camp, Teacher, Bethany High School. Geometry I or II—Room 305, Miss Myrtle Clark, Teacher, Jackson High School.

Mathematics Section (Thurdsay, 3 P.M.—Whittier Junior High School, Room 107): President: Lillith Waggener Beatrice; Vice-President: Bertha Enyeart, Tecumseh; Secretary: Dorothy Swanson, Aurora. Papers: "Devices for Gaining Interest in Ninth Grade Algebra," Miss Pearl Reddish, Adams; "Another Phase of the Individual Difference Problem," W. B. Johns, Teachers College High School, University of Nebraska; "Recreation in Mathematics," Andrewina Stewart, Beatrice Junior High School.

DISTRICT NUMBER TWO AT OMAHA

Mathematics Section (Friday Afternoon 2:00 P.M. Central High School, Room 335): President Dr. J. M. Earl, University of Omaha; Secretary, D. A. Bressler, Uehling.

Papers: 1. Objectives in Secondary Mathematics (20 minutes), Prof. Arthur L. Hill), Peru State Teachers College: Peru; 2. Certain Devices for use in the Teaching of Mathematics (15 minutes), H. A. Campbell, Central High School, Omaha; 3. The College Freshman's Difficulties in Mathematics (40 minutes), Prof. A. R. Congdon, University of Nebraska, Lincoln; Business Meeting.

DISTRICT NUMBER THREE AT NORFOLK

Mathematics (Thursday, 3 P.M. to 5 P.M., Room 304): Mildred Clark, Wayne—Chairman. Address—"Plans for Teaching Quadratics"—Miss Marie Hove, Wayne.

Panel Discussion. Business meeting.

DISTRICT NUMBER FOUR AT GRAND ISLAND

Mathematics Section (Friday Afternoon—1:30, Senior High School—Room 206): President, Martin Johnson Amherst; Vice-President, Eva Phalen, Kearney; Secretary, Roger Harris, Hastings.

Papers: "Fourth Semester Algebra," Clarence Lindahl, Paxton. "Devices and Class Procedures in Algebra," Glen Durflinger, Kearney State Teachers College; Round Table Discussion on "Current Problems Facing the Mathematics Teachers," Led by Professor M. S. Pate, Kearney State Teachers College.

Business Meeting.

DISTRICT NUMBER FIVE AT HOLDREGE

Science and Mathematics Section (Thursday Afternoon, October 26, 3:10 to 4:40, Room 103 Junior-Senior High School Building): Chairman: Paul F. Morse, Benkelman.

Program: "Chemistry in the Small High School," by Superintendent G. C. Olson, Wauneta; "Progress of Science

Teaching," by M. L. Stoner, Minden. Round Table Discussion, led by Frank Wieland. 1. High School Science.

DISTRICT NUMBER SIX AT SCOTTSBLUFF

Mathematics Section (Room 204): President, L. G. Black, Minatare; Secretary, Joe W. Bayer, Gering.

Program: "Securing Individual Interest in Mathematics," Frank Barta, Lisco; "Grading Geometry Proofs," Joe W. Bayer, Gering; "The Common Mistakes in Algebra," A Round Table Discussion. (Each Algebra Teacher was asked to bring a list of mistakes observed.)

Business Meeting.

The above programs from the district meetings of the Nebraska State Teachers Association were collected by Prof. A. L. Hill, Peru State Teachers College, Peru, Neb.

ARTHUR L. HILL
Peru State Teachers College
Peru, Neb.

The Winthrop College Branch of the National Council of Teachers of Mathematics met in Kinard Hall Tuesday, October 24, 1933, with the Vice president, Miss Lillian Henderson, presiding. After a short business session, Miss Almira Johnson played a violin solo. Miss Margaret Bell, instructor of physics at Winthrop College, gave a very interesting talk on "The Fourth Dimension." In conclusion Miss Rebecca Roberts read a poem entitled "Somewhere in the Fourth Dimension."

ELIZABETH HARMON, *Local Editor*

The Spring meeting of The Mathematics Section of the Bay Section of the California Teachers Association was held at The University High School at Oakland on May 20, 1933.

Program: "The Use of the Acoustimeter in measuring noise in the School," by Mr. Clyde Polson, University High School, Oakland, Calif.; "Opportunities in Mathematics at Lowell High School," by Mr. Arthur L. McCarty, Lowell High School, San Francisco, Calif.

The new officers for the year 1933–34 are President, James W. Hoge, University High School, Oakland. Vice-President, S. A. Francis, San Mateo Junior College, San Mateo. Secretary, Miss Nina H. Slewing, University High School, Oakland.

The Eighth Annual Conference of Teachers of Mathematics connected with the University of Iowa Extension Division was held on October 20th and 21st, 1933 at the Old Capitol in Iowa City. The following program was given:

Friday Morning (*North Room, Old Capitol*): H. L. Rietz, Professor of Mathematics, *presiding*.

Addresses: "Teaching Mathematics as a Method of Thinking," W. D. Reeve, Columbia University; "Diophantine Equations," N. B. Conkwright, University of Iowa; "Technique of Instruction in Algebra," W. W. Hart, University of Wisconsin.

Discussion.

Friday Afternoon: E. W. Chittenden, Professor of Mathematics, *presiding*.

Addresses: "The Teaching of Verbal Problems in Algebra," Joseph A. Nyberg, Hyde Park High School, Chicago, Ill.; "Mathematics in an Integrated Program," W. D. Reeve.

Discussion.

A display of equipment and teaching aids for the mathematics of the junior and senior high schools was made in Room 105, University High School.

Friday Evening (*Iowa Memorial Union*): 6:00 P.M. Conference Dinner; Allen T. Craig, University of Iowa, *toastmaster*. Informal Symposium on Tendencies in the Teaching of High School Mathematics.

Saturday Morning (*North Room, Old Capitol*): J. F. Reilly, Professor of Mathematics, *presiding*.

Addresses: "Technique of Instruction in Geometry," W. W. Hart; "Taking Care of Individual Differences," Ruth Lane, University of Iowa; "The Exercises in Plane Geometry," Joseph A. Nyberg.

On Tuesday, October 3, 1933, the Winthrop College Branch of the National Council of Teachers of Mathematics (Rock Hill, S. C.) held its first meeting of the year 1933–34 with the president, Miss Elizabeth Jones, presiding.

The program was as follows: Club song, "If There Were No Mathematics," by the members; business; "A Review of our Mathematical Library," Miss Helen Peden; "humor," Miss Catherine McNally; "Century of Progress Mathematics," Miss Edna Walker.

ELIZABETH HARMON, *Local Editor*

The Mathematics Section of the Northern Oaklahoma Education Association was held at Enid, Oklahoma on Friday morning October 28th at Enid High School.

Program: Addresses: "Mathematics as an Interpreter of the Social and Economic Environment" Loraine White, Blackwell; "Practical Mathematics of Antiquity," H. R. Harold, Tonkawa; "Mathematics Recreations," Anna L. Cowan, Ponca City. Round Table Discussion: "How to Make Mathematics Interesting in the Junior High School," led by Harold Duckett, Enid.

Miss Grace Smith of Enid was chairman of the meeting and Miss Maud Klepzig of Enid was secretary.

The Mathematics Section of the Southern Zone of the New York State Teachers Association held its fall meeting at Central High School in Binghamton on Friday Afternoon October 6th with Miss Helen M. Hibbard of Elmira Free Academy presiding. The following program was presented:

Address: "Approximate Measurements," F. Eugene Seymour, Supervisor of Mathematics, State Education Department Albany. Business Meeting. Address: "The Mathematical Puzzle as a Stimulus to Investigation," Dr. Walter B. Carver, Cornell University.

At the March meeting of the Men's Mathematics Club of Chicago the following program was given: "The Armour Plan," Prof. W. C. Krathwohl; "Interesting Detours in Algebraic Instruction," Prof. S. F. Bibb; "Mathematical Induction," Prof. V. B. Teach.

At the April meeting of the above club the following program was presented: "The Minneapolis Meeting," Mr. C. M. Austin, of Oak Park; "Typical Divisions of Ninth Grade Algebra," Mr. J. J. Urbancek of Lane Technical High School; "Something I am Doing," Messrs. C. W. Cell, O. E. Overn, H. G. Ayre.

These meetings of the Chicago Club were held at the Central Y. M. C. A. Mr. Walter S. Pope, of Morton Township High School, was chairman, Mr. F. W. Runge, of Evanston was Secretary-treasurer, and Mr. J. J. Urbancek of Lane Technical High School was recording secretary.

NEW BOOKS

Mathematical Excursions. By Helen Abbot Merrill. The author, 6, Waban St. Wellesley, Mass., 1933, XIV+145 pp., $1.75.

Mathematical Nuts. By Samuel I. Jones. The author, Life and Casualty Building, Nashville, Tenn., 1932, XII+340 pp., $3.00.

What is the motive force that has pushed mathematics upward and onward all through human history? What power is it that has kept it in the schools since schools began to exist? What resisting energy has thrust aside the pressure of an expanding curriculum that tends to force this one of the older disciplines out of the schools, or to render it a kind of kindergarten subject? It is certainly not mere utility, for the use of percentage is only slightly apparent to a pupil who begins its study; nor is the use of trade discount, or proportion, or the algebraic formula, or the congruence of triangles, or the Law of Sines in trigonometry, or the differentiation of tan x—at least as these parts of mathematics are commonly presented. The days are past for corporal or even mental punishment of a pupil for not learning how to factor a polynomial of the fourth degree, or for failing to show any startling enthusiasm at being told that in mathematical astronomy he will need to know how to use determinants. If utility were the motive force that keeps mathematics advancing, or the adhesive force that makes it stick to the child, or the child to it, then the subject would long since have been banished from the schools. Strikes are not confined to labor unions, and pupils have struck, and struck successfully, for higher intellectual wages, and they will always do so ,led by teachers who have eyes to see. For this reason much of grammar, of geography, of history, and of science which I studied as a child has long since been buried in the vast cemetery of educational theories and dry bones.

No, mathematics, like science and music and art and letters has survived because mankind likes it. More than most other topics in the curriculum it is a game, and it is the game element that should endear it to the pupils of all races and all ages. We are coming to see that subjects which are not made interesting to children are of little value. The day of "practising" on the piano, for example, with no music to entertain the beginner has gone by, or of learning a language by commencing with six months of dull grammar, or of memorizing dates of no interest or importance as an introduction to history. The teacher who cannot lead children or adolescents or adults to enjoy the subject taught, whatever it may be, should either eliminate that subject or eliminate himself.

These two books were written for the purpose of showing a little of the lighter and brighter aspect of that trunk of knowledge which furnishes food to every branch of science and to the arts and industries as well. When a child is asked "How many peaches must be added to three peaches to make seven peaches?" he is given a puzzle and he likes to treat the problem as such. That he is beginning the first step in solving a linear equation concerns him not, nor should it. No pupil ever succeeds in factoring

except he likes to play the game. He will probably make little use of it, but he soon gets a technic which he may need to use,—a matter which has at the time little or no interest for him. That the game may result in something useful sometime does not particularly concern the student; what he likes is the game itself. That it leads somewhere else is the good fortune of mathematics. For this reason the science may well arouse the envious spirit of contract bridge, the cross-word puzzle, poker, or rouge et noir.

Professor Merrill draws upon her long and successful experience as a teacher at Wellesley, and from this experience she is able to make her excursus into the interesting border lines of mathematics and lead the student into new and attractive bypaths in the ever fascinating garden of puzzles.

Why should we be interested (if we are) in a game of solitaire? It is because we like to play the game in which our opponent is a pack of cards. Why should we be interested in finding out why it is that $n(n^2+1)$ is always an even number n being any integer? It is because we like to play the game in which our opponent is a pack of letters which represent whole numbers. It is such games as this that Professor Merrill offers to her readers. She has an attractive lot of flower beds in her garden, or cards in her pack. The cards in her pack are marked "Dividing," "Writing numbers," "Multiplying," "Squares," "Decimals," "True formulas," "Magic squares," "Incommensurables," "Pi," "Geometrical arithmetic," "Oddities," "Equations with many answers," "Drawing a straight line without a ruler," and "The imposssible in mathematics."

How does the game seem? Is it dull? Is it true that the sum of all the factors of 120, including 1 but not 120, is 2×120? It is a nice little game to show that it is, or that it is not, and an easy one. Is it true that every whole number is the sum of two prime numbers, as in the case of $18=7+11$? Probably, but no one has ever yet proved it. This book is full of games of this kind, most of them easy, all of them interesting, some of them hard, one or two impossible of winning. The book ought to be in the hands of all teachers and on the shelves of all high schools and colleges.

Mr. Jones needs no introduction to the teachers of mathematics, here or abroad. His name has been connected with mathematical recreations for a considerable number of years—years which he has made the more enjoyable to teachers and pupils in all types of schools and in hundreds of homes. His latest work, *Mathematical Nuts for Lovers of Mathematics*, might even more appropriately be announced as written "to help make the world love mathematics." It furnishes just the kind of material through the use of which the subject becomes enjoyable to those who (I firmly believe due to poor teaching) have found it dull and uninteresting. The author has separated the "Nuts" in a convenient manner, some "For Young and Old," some for "the Fireside," and others for "the Classroom," the "Math Club," "the Magician," "The Professor," and "the Doctor," with two chapters of "Nuts Cracked for the Weary" and "Nut Kernals," in which the mysteries are revealed. Just why "Mathematics" becomes "Math," while the "Professor" is not exalted to "Prof" and the "Doctor" to "Doc" is a nut for the author to crack. In our schools of education the future teachers of pure English speak of taking course "Ed 120," so perhaps they should speak of this book as "Math Nuts for Lovs of Math."

The book is an exceptionally good collection. It contains problems which have come down to us through the cen-

turies and problems of our own day—nuts that have grown in our own land and those which have come from "the ends of the earth." As a collection of mathematical puzzles of the most usable type it ranks very high, and the school library that does not possess both this and Professor Merrill's work should be put on the black list, and the teacher who does not make use of each should (as the graduates of our schools of letters and of tastes so often say) "Look for another job."

And by the way, How do you prove that the sum of two squares is never divisible by 7 or 11 unless each square is so divisible? (Professor Merrill.) After proving this one, answer this: "If five quarters are tossed at the same time what are the chances that at least four of them will turn up all heads or all tails?" (Mr. Jones.) Why are you going to study either of these? You may say that you won't do so, but you will.

DAVID EUGENE SMITH

Some Influences of the Requirements and Examinations of the College Entrance Examination Board on Mathematics in Secondary Schools of the United States. By Leslie Harper Whitcraft. Bureau of Publications, Teachers College, Columbia University. 1933. x+115 pp. Price $1.50.

Dr. Whitcraft presented a summary of his investigations on the influence of the examinations of the College Entrance Examination Board at the meeting of the National Council of Teachers of Mathematics in Minneapolis on February 25, 1933, this paper appearing in the *Mathematics Teacher* for May. Accordingly, the purpose of this review will be to indicate the arrangement of the material in the longer study without quoting data included in the shorter essay.

The author begins with a brief summary of the place of geometry and algebra in the curricula of various secondary schools indicating that the content of these courses has been subject to change and that this study will seek to deal only with those that may be traced to the influence of the College Entrance Examination Board. There will be a tendency on the part of those who decry the work of the Board to say that these changes would have come about without its agency. Those who view the work of the Board with cordiality will point with pride to the notable changes that have come about under its aegis. The author keeps a judicial attitude, but the reviewer may perhaps been permitted to play the devil's advocate and italicize certain statements, putting comments into parentheses.

The study is five-fold: to consider the growth in the number of candidates for the Board's examinations and the growth in the number of schools from which they come; to analyze examination questions in algebra and in geometry; to analyze texts and courses of study; to assemble the opinions of class room teachers by means of a questionnaire; to study the opinions of experts.

The author classifies the influence of the Board as follows:

The Examinations and Syllabi of the College Entrance Examination Board have:

a. *stimulated better teaching by setting up definite goals.*
b. interfered with better teaching by forcing a curriculum on the schools. (But schools uninfluenced by extramural examinations show about as poor teaching as can be found anywhere.)
c. permitted little freedom in choice of materials and content.
d. retarded creative work. (But creative work goes on. Is this a valid point or is it the defense mechanism of the teacher who thinks he would rather like to do creative work but who finds the examinations a ready excuse for his failure to do so?)

e. *eliminated much of the obsolete material from the Board's requirements . . . and from the mathematics of the secondary schools. . . .*

. forced attention of pupil and teacher on the examinations.

g. caused worry and nervousness to pupil and teacher.

h. standardized the mathematics of the secondary school. (But what state or city would be likely to assemble as competent curriculum committees as those brought together by the College Entrance Examination Board as the lists appear in these pages?)

brought *greater emphasis upon . . . formula, graphs,* simultaneous equations, *. . . the function concept, . . . binomial theorem, logarithms, and verbal exercises.* At the same time they are partly responsible for *less emphasis* being given to two topics the *highest common factor* and the *least common multiple.*

j. Brought *greater emphasis upon . . . locus problems and original.*

k. been chief influence in determining the con" tent of texts. (This would seem to be partly through the conviction of the authors and respect for the ability of the men who made the syllabus and partly perhaps through prudence.)

l. influenced state and city courses of study.

m. forced these things into the curriculum for the benefit of the few students in a class going to college regardless of the small proportion which these are of the whole.

How far these things may be traced to the College Entrance Examination Board is debatable. Would the additions and omissions have been accomplished under the leaven of the National Committee's work unaided by the Board? How far is the influence of the Board really detrimental? The author does not say. He merely presents his findings leaving the reader to judge them. But to continue in the rôle of the devil's advocate, the reviewer would like to call the reader's attention to the fact that the quickest way to get a subject into the curriculum in mathematics is to incorporate it in one of the examinations of the College Board. While these topics would ultimately creep into the curriculum, their presence in the examinations insures their appearance in subsequent years in the class room.

VERA SANFORD

THERE is no use in railing at America, or expecting her to take her proper position in this world, unless each of us as citizens makes an effort to make his own life worth while on this basis of values which are not wholly material, and strives to make the life of this nation truely great and not merely a standard of living at so many thousands a year.

—JAMES TRUSLOW ADAMS

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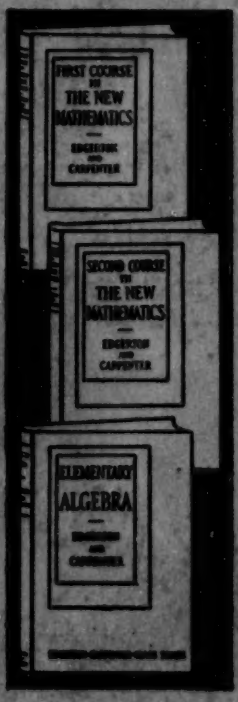

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